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Particle production spectra and the internal structure of the hadrons

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Abstract. Longitudinal momentum distributions of produced pions from hadron–hadron collisions were derived from the quark–boson hadron model in which the hadron is a bound system of quarks (three for a nucleon, two for a meson) and a number of fluctuating charged and neutral bosons. The model was obtained previously by the authors from the study of the inelastic form factors, or the structure functions, in eN, νN and $\bar{\nu}$N scattering. The momentum distributions derived here with no free parameter fit through the five-decade measured cross sections for pp interactions and reproduce the asymmetric spectra observed in π^-p, π^+p and K^+p interactions.

1. Introduction

This paper presents further evidence for the proposition (Wang 1969a, b, c and d) that a hadron is composed of a number of smaller subunits which make independent contributions to the matrix elements in high energy interactions‖. That proposition has (Wang and Lin 1971, 1972a and b) been made more specific by assuming that in a nucleon those subunits are the three quarks (two in a meson) and a number of tightly bound quark–antiquark pairs, forming thus $3n_1$ bosons. The factor 3 means that we assume that in fluctuations, three bosons, of charge $+1$, -1 and 0, are formed or destroyed together on the average and n_1 is the number of these fluctuating boson triplets. (At the energies here considered, they must be mostly pions.) For computing the momentum distributions, we note that these bosons move as individual particles. For computing interactions, however, each boson is considered as a quark–antiquark pair.

This assumption of independent particle production from those subunits leads one to predict a Poisson distribution for the number n_c of the secondary *charged pairs produced* and the preservation of identities of the two incoming hadrons. Such a Poisson distribution has indeed been found (Wang 1969a to g, 1970a) in all experiments in which the multiplicity of production in hadron–hadron collisions has been studied. In $\bar{p}$p and $\bar{p}$n annihilation, the distribution is modified by the presence of three individual antiquarks that carry the baryon number in the antinucleon. Here again the modified distribution has been found (Wang 1969e, f and g).

In this paper, we derive from this model the longitudinal centre of mass (CM) momentum distributions for pions produced from $\pi^{\pm}$p, K^+p and pp interactions.

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‖ Later versions of this model, employed by other authors, have become known as the 'parton model'.

As will be seen in §§ 3 and 4, in all of these interactions it is found that the mean 'internal' multiplicity $\bar{n}_1$ is given by the mean 'external' charged-pair multiplicity $\bar{n}_c$ which is determined from the experiment itself. Thus, the fits to the longitudinal spectra measured do not contain any free parameters. It is remarkable that the momentum distributions so calculated fit through the five decades of the measured cross section for pp interactions, and give the left and the right branch of the asymmetric spectra observed in $\pi^{\pm}$p and K^{+}p interactions. These, together with the excellent results obtained previously by us (Wang and Lin 1971, 1972a and b) for eN, νN and $\bar{\nu}$N interactions, in our opinion, are strong evidence for the fine structure of hadrons we picture. That the constituent quarks have charges of $\frac{2}{3}$ and $\frac{1}{3}$ is strongly supported by our successful simultaneous fits to the inelastic form factors, or the structure functions νW_2 and $2MW_1$, for ep scattering.

2. Particle 'production cross sections' and nucleon–meson fine structure functions

Consider a hadron with a number of subunits. At high energies, each of these will have a momentum in the longitudinal direction a fraction x of that of the incoming hadron, and the momentum distribution will be given by

$$\frac{\mathrm{d}N}{\mathrm{d}x} \propto \sum_n P(n)F_n(x) \tag{1}$$

where $P(n)$ is the probability for the hadron to be in a quantum state with n subunits and $F_n(x)$ is the distribution function of x when the hadron is in such a state:

$$n = \int_0^1 F_n(x)\,\mathrm{d}x. \tag{2}$$

We assume as before that $F_n(x)$ is given by the one dimensional phase space function

$$F_n(x) \equiv n(x) = nf_n(x) = n(n-1)(1-x)^{n-2} \tag{3}$$

where $f_n(x)$ is the normalized probability density of the variable x and $F_n(x)$ was previously called $n(x)$.

For the 3-quark–boson composite nucleon we assume, the bosons have the meson quark contents p′n̄′, n′p̄′, (p′p̄′ − n′n̄′)/$\sqrt{2}$, and are 1:1:1 in number on the average because charge conservation and symmetry is assumed for the three charge states. Thus in a state with n_1 boson triplets, $n = 3n_1+3$, and the x distribution of these $3n_1$ bosons, from equation (3), is

$$F_{3n_1}(x) = (3n_1+3-3)(3n_1+2)(1-x)^{3n_1+1}.$$

Since charged pion pairs (and the neutral pions) are produced from these production cells or ylon subunits and follow the poissonian W^{I} distribution (Wang 1969a to g, 1970a, Elbert *et al* 1970, Jones *et al* 1970) it is to be expected that these bosons will be poissonian in number and thus $P(n)$ will be poissonian in n_1, as we assume for the electron and the neutrino case:

$$P(n) = W^{\mathrm{I}}(n_1) = (\bar{n}_1^{n_1}/n_1!)\exp(-\bar{n}_1) \tag{4}$$

where $W^{\mathrm{I}}(n_1)$ is also the usual Poisson probability function with the mean value $\bar{n}_1$ omitted from the symbol for clarity.

The 'production cross section' of a pion of one sign from such a composite *nucleon*, from equation (1), will then be

$$\left(\frac{d\sigma}{dx}\right)_{\text{nucleon}} \propto \sum_{n_1=1}^{\infty} W^1(n_1)\frac{3n_1+3-3}{3}(3n_1+2)(1-x)^{3n_1+1}. \tag{5}$$

The summation is from $n_1 = 1$, the first excited state corresponding to production.

The production from a *meson* will be similar except that it consists of *two* quarks and a collection of bosons of similar properties. Therefore in a quantum state with n_1 boson triplets, the number of subunits $n = 3n_1+2$. Production of a pion of one sign from such a composite meson will then be

$$\left(\frac{d\sigma}{dx}\right)_{\text{meson}} \propto \sum_{n_1=1}^{\infty} W^1(n_1)\frac{3n_1+2-2}{3}(3n_1+1)(1-x)^{3n_1}. \tag{6}$$

It is convenient to define two sets of functions H_i and $J_i = H_i/(1-x)$ as the 'fine structure function' of the nucleon and the meson, as follows ($i = 0, 1, 2$):

$$H_i = \sum_{n_1=1}^{\infty} W^1(n_1)n_1^i(1-x)^{3n_1+1}. \tag{7}$$

Equations (5) and (6) are then simply

$$\left(\frac{d\sigma}{dx}\right)_{\text{nucleon}} \propto 3H_2+2H_1 \tag{8}$$

$$\left(\frac{d\sigma}{dx}\right)_{\text{meson}} \propto 3J_2+J_1. \tag{9}$$

Explicit expressions for H_i, and hence J_i, are

$$H_0 = (1-x)\exp\{\bar{n}_1(1-x)^3\}\,e^{-\bar{n}_1} - (1-x)e^{-\bar{n}_1}$$

$$H_1 = (1-x)\bar{n}_1(1-x)^3\exp\{\bar{n}_1(1-x)^3\}\,e^{-\bar{n}_1}$$

$$H_2 = (1-x)\{\bar{n}_1^2(1-x)^6+\bar{n}_1(1-x)^3\}\exp\{\bar{n}_1(1-x)^3\}\,e^{-\bar{n}_1}. \tag{10}$$

3. Pion spectra from π^-p, π^+p and K^+p interactions

Figure 1 shows the distribution of π^- and π^+ mesons produced from the π^-p interaction at 25 GeV/c in the CM backward and forward hemispheres respectively as measured by the Wisconsin group (Elbert *et al* 1971). This reaction as we did before may be represented by the following equation:

$$\pi^- + p \rightarrow C + D + n_c(\pi^+ + \pi^-) + n_0\pi^0 \tag{11}$$

where C and D are (1) π^- and p, (2) π^- and (n + π^+), or (3) π^0 and n, respectively, for these three production channels and $n_c, n_0 = 0, 1, 2, 3 \ldots$ are the numbers of the charged pion pairs and the neutral pions emitted from the ylon production cells of the target proton and the projectile π^-, according to the production and the subunit structure of the hadrons we picture.

Since the momenta of these 'truly produced' pions (a term introduced at the 1970 Athens International Conference and at a seminar in 1969 at Wisconsin (Wang 1970b and c)) in the respective frame of their parent hadrons are small as shown

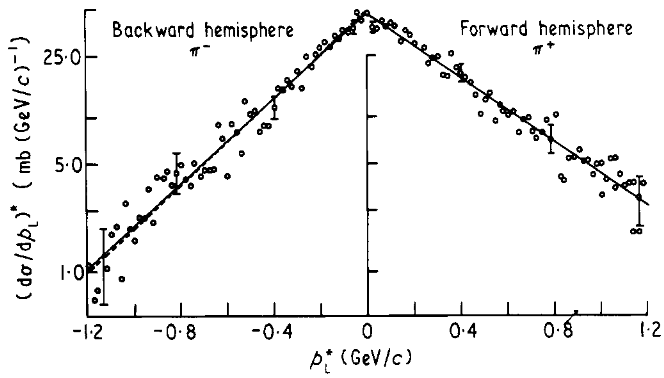

Figure 1. Asymmetric 'production cross section' or spectrum of the CM longitudinal momentum p_L^* of the 'produced' pions from π^-p interactions at 25 GeV/c. The full 'lines' are the spectrum calculated from our hadron model. The right branch coincides with the empirical exponential fit by the original experimenters, and the left branch shows the small curvature contained in the theoretical expression as compared to the straight-line (broken) empirical exponential fit. The experimental points and the empirical exponential fits are from Elbert *et al* (1971).

by the small mean value ($\simeq 0{\cdot}4$ GeV/c) of their transverse component, those pions emitted from the projectile π^- are to be found in the forward CM hemisphere, and those from the target proton, in the backward hemisphere. Hence the π^- mesons with negative p_L^* are the π^- of those charged pairs contained in the term $n_c(\pi^+ + \pi^-)$ that are produced from the target proton, and the π^+ with positive p_L^*, the π^+ in the same term that are produced from the projectile π^-.

In our picture of production, the two incoming hadrons will preserve their identities in a collision, and in the target frame, the projectile hadron becomes the 'leading' particle observed in cosmic ray jets. This preservation of identities of the two incoming hadrons, and hence the distinction between the 'truly produced' particles or the 'produced' particles, $n_c(\pi^+ + \pi^-) + n_0\pi^0$, and the 'leading' ones, C and D, are not assumptions but were deduced from a great number of high energy collision experiments as was expressed by the term $(n_s - \alpha)$ in the $W^{\mathrm{I}}_{n_s}$ distribution (and also the $W^{\mathrm{II}}_{n_s}$ distribution) deduced by us for the total number n_s of all the charged secondaries emerging from a collision (Wang 1969a to g, 1970a):

$$W^{\mathrm{I}}_{n_s} = \frac{\langle (n_s - \alpha)/2 \rangle^{(n_s - \alpha)/2}}{\{(n_s - \alpha)/2\}!} \exp\left\langle \frac{-(n_s - \alpha)}{2} \right\rangle = W^{\mathrm{I}}(n_c) \tag{12}$$

where

$$n_c = \tfrac{1}{2}(n_s - \alpha) \tag{13}$$

is the number of charged pairs produced, $\alpha = 2$ or 1, the number of charge of the two incoming hadrons, and $W^{\mathrm{I}}(n_c)$, the symbol used in equation (4) for the usual Poisson probability function (here with n_c as its argument).

Thus, we fitted the left branch of the spectrum with equation (8) and the right branch, with equation (9). The mean charged pion pair multiplicity which we call $\bar{n}_c$, is 1·45 in this experiment. As was shown previously by us (Wang 1970b and c), $\pi^{\pm}$p and pp collisions at the same CM available energies have nearly the same values of $\bar{n}_c$ (up to

30 GeV/c laboratory incoming momentum beyond which there are not enough experimental data to compare). Hence the average number of charged pions produced from the incoming pion as a first approximation may be taken to be the same as that from the incoming proton. The mean charged-pair multiplicity for either the target or the projectile in this experiment is thus about 1·45/2 = 0·725. We assign this to be the mean multiplicity $\bar{n}'_1$ of the boson triplets associated with each component quark of the proton or the π^- projectile, so that the total 'internal' multiplicities $\bar{n}_1$ in equations (8) and (9) are

$$\bar{n}_1 = 3\bar{n}'_1 = 2{\cdot}18 \qquad \text{for target proton}$$
$$\bar{n}_1 = 2\bar{n}'_1 = 1{\cdot}45 \qquad \text{for projectile } \pi^-. \qquad (14)$$

The two full lines of figure 1 are the two curves of the CM longitudinal momentum $p_L^* = xp_0^*$ (p_0^* is the CM momentum of the incoming hadron) thus calculated for the backward π^- and the forward π^+ respectively. They coincide practically with the empirical exponential fits by the original experimenters for the range of p_L^* covered.

Similar perfect fits to data were obtained for π^+p interactions at 7 and 18·5 GeV/c and K^+p at 12·7 GeV/c measured by the Notre Dame group (Biswas *et al* 1971) and the Rochester group (Stone *et al* 1971). These are shown in figures 2, 3 and 4 where the experimental distributions are for the π^- of the reactions

$$\pi^+ + p \rightarrow \pi^- + \text{anything}$$
$$K^+ + p \rightarrow \pi^- + \text{anything}$$

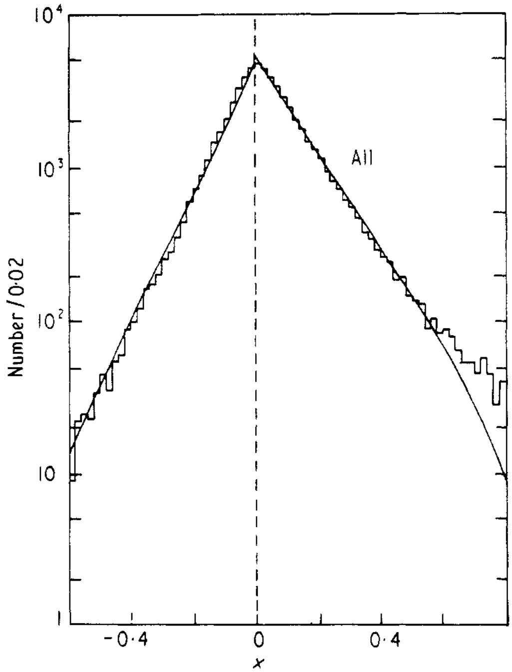

Figure 2. Asymmetric 'production cross section' or spectrum of the π^- produced from π^+p interactions at 18·5 GeV/c. The full curves are the spectrum calculated from our hadron model. The measured spectrum is from Biswas *et al* (1971). Here $x = p_L^*/p_0^*$.

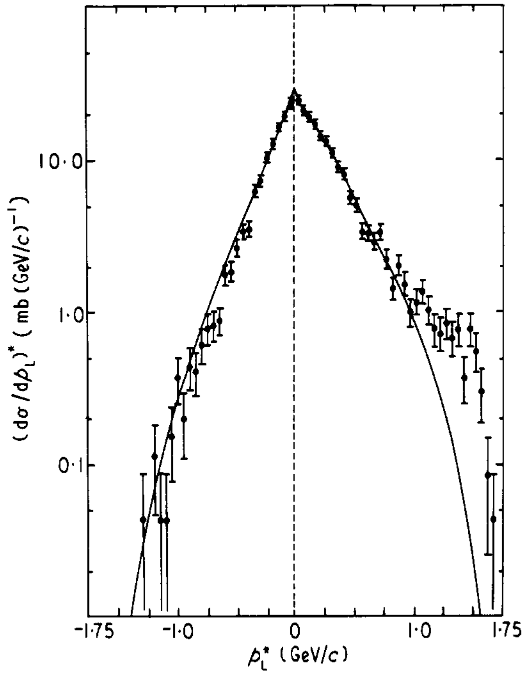

Figure 3. Asymmetric 'production cross section' or spectrum of the π^- produced from π^+p interactions at 7 GeV/*c*. The full curves are the spectrum calculated from our hadron model. The structure on the right branch of the measured spectrum is due to resonance production. The experimental points are from Stone *et al* (1971).

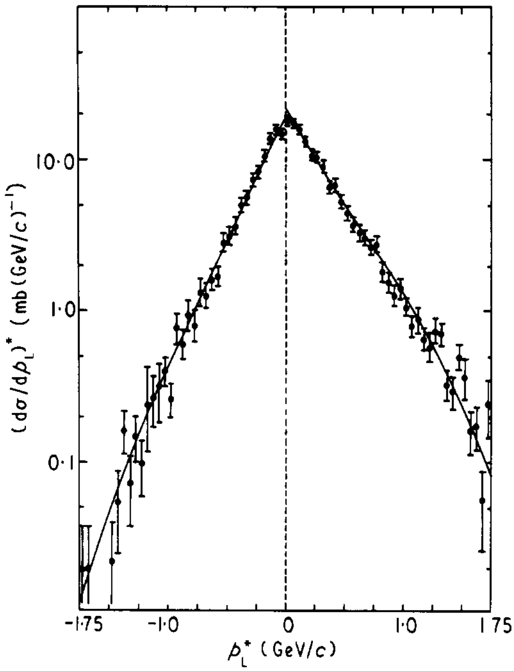

Figure 4. Asymmetric 'production cross section' or spectrum of the π^- produced from K^+p interactions at 12·7 GeV/*c*. The full curves are the spectrum calculated from our hadron model. The experimental points are from Stone *et al* (1971).

and hence they are the spectra of the pions produced from the target proton and the projectile π^+ and K^+ meson for the left (negative x) and the right (positive x) branch respectively. (We note the structure due to resonance production shown in the right branch of the measured spectra for $p_L^* \gtrsim 1.0$ GeV/c. This is seen especially clearly from figure 3 where the whole range of x from 0 to 1 has been covered by the experiment —here $p_0^* = 1.75$ GeV/c.)

The external multiplicities $\bar{n}_c$ for these three experiments were calculated from an empirical formula obtained previously by us for pp and $\pi^{\pm}$p interactions (Wang 1970a, b and c):

$$\langle n_s - 2 \rangle = a \lg \epsilon_{cm} + b \tag{15}$$

with $a = 2.77 \pm 0.16$, $b = 0.19 \pm 0.07$, for the CM available energy ϵ_{cm} lying between 1 and 5 GeV, as given by the least squares fit to the data points in that energy range.

Exactly similar perfect fits to the asymmetric π^- spectra measured by the UC Davis group for K^+p interactions at 11.7 GeV/c (Ko and Lander 1971) were obtained.

Fits to the spectra of the 'truly produced' or the 'produced' particles for fixed prong number channels are equally successful with our hadron model and will be discussed elsewhere.

4. Pion spectra from pp interactions

The spectrum of pions produced from either proton in pp interactions again is given by equation (8). Figure 5 shows the measured spectrum of π^- produced from pp collisions at 30 GeV/c (Anderson *et al* 1967) and the parameterless fit of equation (8) with $\bar{n}_1 = 2.2$. Here $\epsilon_{cm} = 5.66$ GeV, a value almost the same as the Wisconsin π^-p experiment at 25 GeV/c, that is, 5.84 GeV. Thus we use the value of $\bar{n}_1 = 3\bar{n}_1' = 2.18$ as given by the Wisconsin π^-p experiment, rounded off to 2.2. The fit is perfect for the whole range of p_L^* covered by the experiment.

A similar perfect fit to data is also obtained for the Scandinavian hydrogen bubble chamber pp experiment (Bøggild *et al* 1971) at 19.2 GeV/c, as shown by the full curve

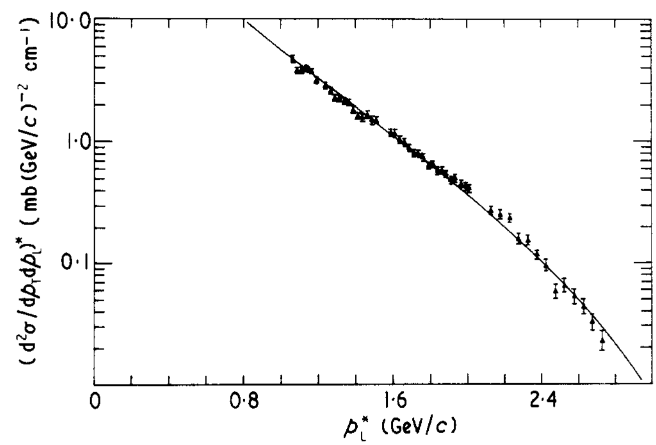

Figure 5. 'Production cross section' or spectrum of the π^- produced from pp interactions at 30 GeV/c. The full curve is the spectrum calculated from our hadron model. The experimental points are from Anderson *et al* (1967) with $p_T^* = 0.18 \pm 0.02$ GeV/c.

of figure 6 which practically coincides with the empirical fit by the original experimenters. This experiment covers a range of x from about 0·4 down to 0·007. Thus, like the πp and Kp interactions discussed in § 3 above, our formula fits pp interaction data from $x \simeq 0$ for practically the whole range of x.

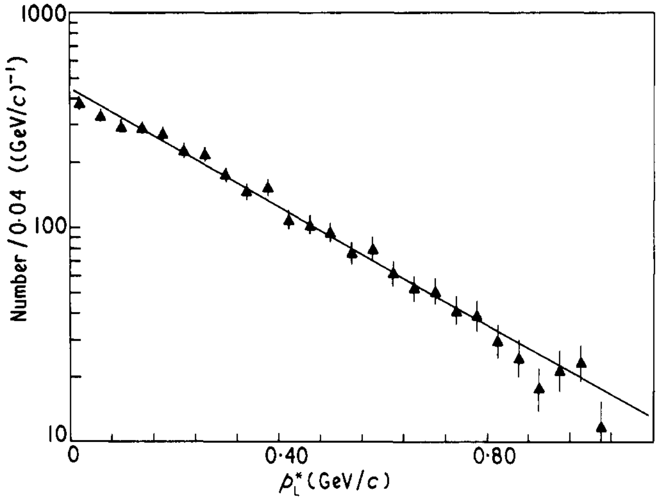

Figure 6. Spectrum of the π^- produced from pp interactions at 19·2 GeV/c. The full 'line' is the spectrum calculated from our hadron model. It coincides practically with the empirical fit by the original experimenters. The experimental points are from Bøggild *et al* (1971). They cover a range of $x = p_L^*/p_0^*$ from 0·007 to about 0·4.

Recently Allaby *et al* (1971) have measured the π^- spectrum from pp collisions at 19·2 GeV/c for $x = 0{\cdot}24$ to 0·92, and have plotted this in the form of

$$\rho_{\pi^-}(x, p_T) = \frac{2}{\sigma_T} \frac{E^*}{\pi p_0^*} \frac{d^2\sigma}{dp_T^* \, dx} \tag{16}$$

with p_T^* extrapolated to zero with an uncertainty of about 10–15% for comparison with π^- spectra obtained at other energies, as shown in figure 7. Since ρ_{π^-} has been shown (Bøggild *et al* 1971) to quite a high degree of accuracy to be factorizable

$$\rho_{\pi^-} = F(p_T)G(x) \tag{17}$$

our formula of equation (8) when multiplied by E^*/p_0^* and normalized should give $\rho_{\pi^-}(x, p_T = 0)$. The full curve marked 19·2 GeV/c in figure 7 is such a plot with $\bar{n}_1 = 1{\cdot}5$ as calculated from equation (15) and checked with the experimental value of $\langle n_s - 2 \rangle = 1{\cdot}01$ measured by the Scandinavian group.

In the same figure we have also extended the plot of Allaby *et al* for 19·2 GeV/c with the Scandinavian HBC data down to $x = 0{\cdot}007$ using the original measured values of $d^2\sigma/dp_T \, dp_L^*$ shown in figure 5 of Bøggild *et al* (1971), and weighted with E^*. (We believe the factor E^* is missing in the original plot by Allaby *et al* of the Scandinavian data.)

The full curve marked 30 GeV/c in figure 7 is the theoretical curve calculated in the same way from equation (8) with $\bar{n}_1 = 2{\cdot}2$ for this momentum *and the same normalization*

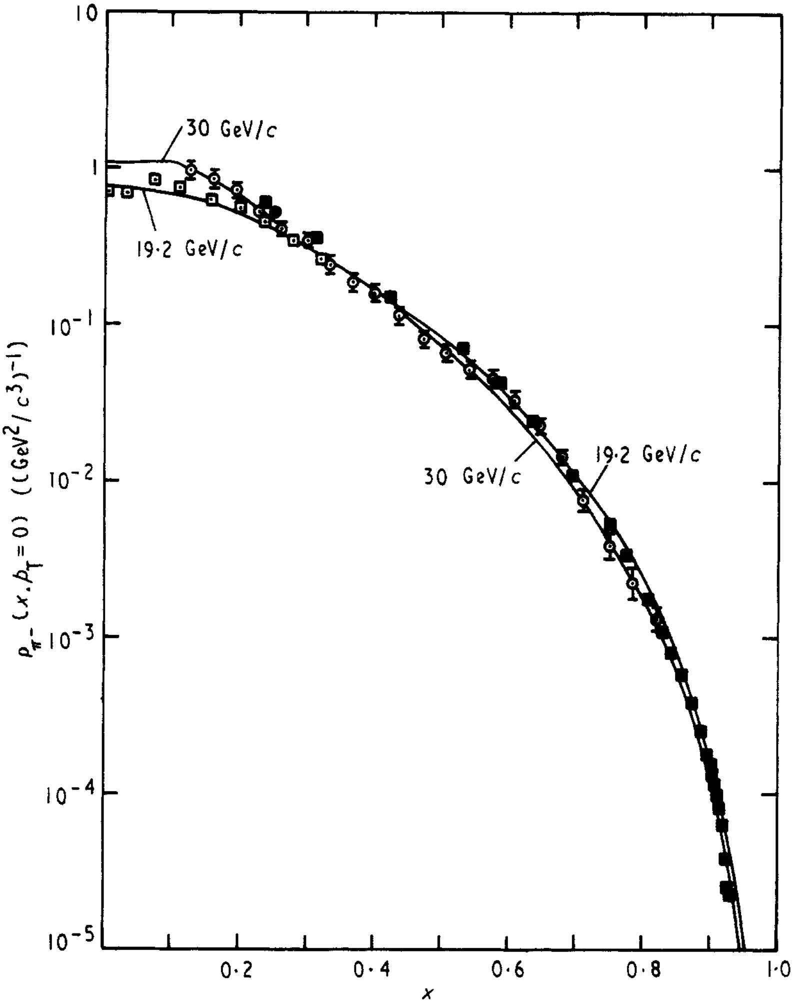

Figure 7. Comparison of production spectra of the 'produced' pions (here π^-) from pp interactions at different energies. The full curves are the spectra given by our hadron model for 19·2 and 30 GeV/c. The experimental plots are from Allaby *et al* (1971), with the 19·2 GeV/c plot extended by us with the Scandinavian data taken from figure 6. ● p_0 = 12·5 GeV/c, 0 mr, Akerlof *et al* 1971 *Phys. Rev.* D 3 645; ■ p_0 = 19·2 GeV/c, 70, 12·5 mr, Allaby *et al* (1971); ⊡ p_0 = 19·2 GeV/c, 0 mr, Bøggild *et al* (1971); ⊙ p_0 = 30 GeV/c, 67·15 mr, Anderson *et al* (1967).

constant fixed by the 19·2 GeV/c *full curve.* It is remarkable that both curves give excellent no-parameter fits to data covering five decades of 'cross section' for the whole range of x, and with the normalization of one determining the other.

We note here that equations (14) which give the internal multiplicity $\bar{n}_1$ from the external multiplicity $\bar{n}_c$ observed in hadron–hadron collisions, not only mean that the number of bosons associated with each component quark is the same in a collision on the average but also indicate that production may come mainly from one component quark of each colliding hadron.

5. Conclusion

In conclusion, the perfect no-parameter fits to the momentum spectra of particles (pions) produced from hadron–hadron collisions with the momentum distributions derived from our quark–boson model which was deduced originally from the study of the inelastic form factors in eN, νN and $\bar{\nu}$N interactions, make us believe that the fine structure of the composite hadrons we picture is very close to reality.

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